

Symmetry breaking of the symmetric left–right model without a scalar bidoublet

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Abstract. By invoking the existence of a general $O(2)$ symmetry, a minimal left–right symmetric model based on the gauge group $G = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ is shown to require the existence of only two physical Higgs bosons. The lighter Higgs is predicted to have a small mass which could be evaluated by standard perturbation theory. The fermionic mass matrices are recovered by insertion of ad hoc fermion–Higgs interactions. The model is shown to be undistinguishable from the standard model at the currently reachable energies.

Left–right (LR) symmetric extensions of the standard model (SM) have been extensively studied since 1974 when they were first discussed [1–3]. Some years ago the quantization and renormalization have been worked out by Duka et al. [4], who also gave an extended literature on the subject. More recently radiative corrections have been considered in order to discuss the phenomenological predictions of such models [5].

LR models are based on the gauge group G , where

$$G = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}, \quad (1)$$

and have the remarkable merit of predicting a phenomenology which is basically undistinguishable from the standard model at the currently reachable energies. However, for the same reason the lack of any experimental evidence makes the choice of the symmetry group – namely LR versus standard – a purely aesthetic matter. From this point of view LR models have quite some advantages, as the observed LR asymmetry is explained by low energy breaking of the symmetry and does not require to be inserted by hand as it is the case for the standard model. Moreover the $U(1)$ generator gets a physical interpretation as the $B - L$ number. On the other hand in their “minimal” version LR symmetric models require the existence of many new particles: the most disturbing ones are the ten physical Higgs particles required (four charged and six real neutral bosons) to be compared with only one physical Higgs boson predicted by the SM.

In this paper it is shown that the proliferation of Higgs particles is not necessary and can be avoided in a truly minimal version of the LR symmetric model.

Any viable gauge model for electro-weak interactions must give an answer to two quite different problems:

- (i) the breaking of symmetry – from the full gauge group to the electro-magnetic abelian group $U(1)_{em}$ – which gives a mass to the gauge bosons and thus explains the known structure of the weak interactions;
- (ii) the mass matrices for fermions.

Even in the standard model these two problems are addressed in different independent steps, and with a different degree of success. In fact, while the first problem is given a full and satisfactory solution, the mass generation for fermions is only described by the ad hoc insertion of a handful of coupling parameters. Thus the mass hierarchy problem has not found any genuine solution. Moreover this second aspect is strongly related to the nature of the Higgs sector which is still unexplored from the experimental point of view. In LR models, the proliferation of Higgs particles is a direct consequence of the very complicated structure of the Higgs sector, required in order to explain the spontaneous breaking of LR symmetry. Unless the Higgs bosons are composite objects, as predicted by top [6] or neutrino [7] condensation models, the prospect of such a large number of elementary fields is not very attractive.

By invoking the existence of a general symmetry, we show that a truly minimal LR model, based on the gauge group G , only requires the existence of two physical neutral scalar Higgs bosons in order to address point (i). Point (ii) remains open to several quite different descriptions which are compatible with the proposed symmetry breaking path. The ad hoc insertion of free coupling parameters between fermions and the two Higgs fields still allows for a full description of the fermion mass matrices, at the extra cost of inserting non-renormalizable terms in the lagrangian. Of course this choice would not give any real answer to the mass hierarchy problem, which remains unsolved.

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The LR symmetric lagrangian is the sum of a fermionic term \mathcal{L}_f , a Yang–Mills term for the gauge bosons \mathcal{L}_{YM} , a Higgs term \mathcal{L}_H and eventually the Higgs–fermions interaction term \mathcal{L}_{int} .

In order to deal with point (i) above, we need to specify \mathcal{L}_H :

$$\mathcal{L}_H = -\frac{1}{2}|D_L^\mu \chi_L|^2 - \frac{1}{2}|D_R^\mu \chi_R|^2 + V(\chi_L, \chi_R), \quad (2)$$

where the covariant derivative D_a^μ is defined according to

$$D_a^\mu = \left(\partial^\mu - ig_a \mathbf{A}_a^\mu \mathbf{T}_a + i\tilde{g} B^\mu \frac{Y}{2} \right), \quad a = L, R. \quad (3)$$

\mathbf{T}_L , \mathbf{T}_R and Y are the generators of $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively, with couplings $g_L = g_R = g$ and \tilde{g} . As usual the electric charge is given by $Q = T_{L3} + T_{R3} + Y/2$. The Higgs fields χ_a are doublets

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}, \quad (4)$$

with the transformation properties

$$\chi_L \equiv (2, 1, 1), \quad \chi_R \equiv (1, 2, 1). \quad (5)$$

A standard \mathcal{L}_{YM} is considered for the seven gauge fields \mathbf{A}_L^μ , \mathbf{A}_R^μ and B^μ . Fermions are described by doublets of spinors ψ_L , ψ_R with the transformation properties

$$\psi_L \equiv (2, 1, B-L), \quad \psi_R \equiv (1, 2, B-L). \quad (6)$$

Their lagrangian term \mathcal{L}_f is as follows:

$$\mathcal{L}_f = -\bar{\psi}_L \gamma_\mu D_L^\mu \psi_L - \bar{\psi}_R \gamma_\mu D_R^\mu \psi_R. \quad (7)$$

The lagrangian $\mathcal{L} = \mathcal{L}_f + \mathcal{L}_{\text{YM}} + \mathcal{L}_H$ is fully symmetric for LR exchange. Moreover we notice that for a vanishing coupling $g, \tilde{g} \rightarrow 0$, and neglecting the contribution of $V(\chi_L, \chi_R)$, the free part of $\mathcal{L}_f + \mathcal{L}_H$ has a global $O(2)$ symmetry, as it is invariant under rotations in the LR plane. We may define doublets of doublets according to

$$\Phi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad (8)$$

and the free lagrangian reads

$$\mathcal{L}_f + \mathcal{L}_H = -|\partial_\mu \Phi|^2 - \bar{\Psi} \gamma^\mu \partial_\mu \Psi, \quad (9)$$

which is invariant under $\Phi \rightarrow \Phi' = R(\theta)\Phi$ and $\Psi \rightarrow \Psi' = R(\theta)\Psi$, where $R(\theta) \in O(2)$ is the 2×2 rotation matrix of angle θ . While the physical meaning of this continuous global symmetry is not evident, we may assume that the full lagrangian should be $O(2)$ invariant in the limit $g, \tilde{g} \rightarrow 0$ of no gauge coupling. According to such an assumption the Higgs potential $V(\chi_L, \chi_R)$ should be invariant under rotations in the LR plane. This symmetry makes the potential V a function of the rotational invariant field ρ :

$$\rho^2 = \chi_L^2 + \chi_R^2, \quad (10)$$

and this is going to hold even in the presence of finite gauge interactions which break the $O(2)$ symmetry. Provided that $V(\rho)$ has a minimum for a non-zero expectation value of ρ , according to (10) the minimum is going to be on a circle in the χ_L, χ_R plane. For the real vacuum, the actual value of θ is only determined by chance. Thus, the existence of the global $O(2)$ symmetry would give a natural path towards the LR symmetry breaking.

If the gauge interactions are allowed to break the $O(2)$ symmetry then infinite renormalizations of the $O(2)$ breaking terms would spoil the symmetry of the scalar potential. Thus we must assume that the quartic terms in the potential are set to zero by some unspecified physics in the high energy theory. It is evident that this assumption turns out to be the weak point of the model: there is no other way to justify the existence of the global $O(2)$ symmetry. However, the main message of the present paper is that the existence of this symmetry is the only way to recover the left–right symmetry breaking from a fully symmetric lagrangian and without any Higgs bidoublet. Models with a soft breaking of the lagrangian symmetry have been proposed since 1975 [8], but they cannot be regarded as truly spontaneous symmetry breaking mechanisms as the symmetry is broken by insertion of “ad hoc” mass terms in the lagrangian. In other words these models do not provide a way to understand why the real vacuum is not left–right symmetric unless we accept that the lagrangian is not symmetric. As the great appeal of left–right models is due to the full left–right symmetry of the lagrangian, it would not be desirable to spoil this symmetry even with mass terms. Thus the existence of a global $O(2)$ symmetry seems to be the only way to avoid any Higgs bidoublet with a fully left–right symmetric lagrangian.

The physical content of the theory becomes more evident in unitarity gauge. Accordingly we set $\chi_a^+ = 0$ and choose χ_a^0 real. The covariant derivative of χ_a reads

$$D^\mu \chi_a = \begin{pmatrix} -i\frac{g}{\sqrt{2}} W_a^{-\mu} \chi_a^0 \\ \left(i\frac{g}{2} A_{a3}^\mu + i\frac{\tilde{g}}{2} B^\mu + \partial^\mu \right) \chi_a^0 \end{pmatrix}, \quad (11)$$

where $W_a^\pm = \frac{1}{\sqrt{2}}(A_{a1} \pm iA_{a2})$. For a non-zero expectation value of ρ we get the following vacuum expectation values for the Higgs fields:

$$\langle \chi_L^0 \rangle = v = \rho \sin \theta; \quad \langle \chi_R^0 \rangle = w = \rho \cos \theta, \quad (12)$$

and we assume that by chance θ is very small ($v \ll w$). Insertion of (11) in the lagrangian (2) yields the mass matrix for the gauge bosons.

The charged W_L^\pm and W_R^\pm are decoupled with masses

$$M_{W(L)} = \frac{gv}{2}, \quad M_{W(R)} = \frac{gw}{2}. \quad (13)$$

Thus the angle θ determines the mass ratio $\tan \theta = M_{W(L)}/M_{W(R)}$. For the neutral gauge bosons B^μ , A_{L3}^μ and A_{R3}^μ we get the mass matrix M^2

$$M^2 = \frac{1}{4} \begin{pmatrix} \tilde{g}^2(v^2 + w^2) & g\tilde{g}v^2 & g\tilde{g}w^2 \\ g\tilde{g}v^2 & g^2v^2 & 0 \\ g\tilde{g}w^2 & 0 & g^2w^2 \end{pmatrix}. \quad (14)$$

There is a vanishing eigenvalue for the electromagnetic unbroken $U(1)$ eigenvector

$$A^\mu = \frac{e}{g} A_{L3}^\mu + \frac{e}{g} A_{R3}^\mu - \frac{e}{\tilde{g}} B^\mu, \quad (15)$$

while the non-vanishing eigenvalues are given by a small value,

$$M_Z^2 = \frac{g^2 v^2 (g^2 + 2\tilde{g}^2)}{g^2 + \tilde{g}^2} + \mathcal{O}(v^2/w^2), \quad (16)$$

and a large one,

$$M_{Z'}^2 = \left(M_{W(L)}^2 + M_{W(R)}^2 \right) (1 + \tilde{g}^2/g^2) - M_Z^2. \quad (17)$$

The corresponding eigenvectors are

$$\begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} -e/\tilde{g} & e/g & e/g \\ g/D_L & \frac{\tilde{g} \cos^2 \theta_L}{D_L \sin^2 \theta_L} & \frac{\tilde{g} \cos^2 \theta_L}{D_L (\tan^2 \theta - \cos^2 \theta_L)} \\ g/D_R & \frac{\tilde{g} \cos^2 \theta_R}{D_R (\tan^{-2} \theta - \cos^2 \theta_R)} & \frac{\tilde{g} \cos^2 \theta_R}{D_R \sin^2 \theta_R} \end{pmatrix} \times \begin{pmatrix} B \\ A_{3L} \\ A_{3R} \end{pmatrix}, \quad (18)$$

where

$$D_L = \sqrt{g^2 + \tilde{g}^2 (\tan^{-4} \theta_L + \cos^4 \theta_L / (\tan^2 \theta - \cos^2 \theta_L)^2)}, \quad (19)$$

$$D_R = \sqrt{g^2 + \tilde{g}^2 (\tan^{-4} \theta_R + \cos^4 \theta_R / (\tan^{-2} \theta - \cos^2 \theta_R)^2)}, \quad (20)$$

and the Weinberg angles θ_L and θ_R are defined according to $\cos \theta_L = M_{W(L)}/M_Z$, $\cos \theta_R = M_{W(R)}/M_{Z'}$.

The transformation matrix (18) can be inverted, and neglecting contributions of order $\mathcal{O}(v^2/w^2)$ and the heavy Z' we find

$$B^\mu = -\frac{e}{\tilde{g}} A^\mu + \frac{g}{\tilde{g}} \tan \theta_L \sin \theta_L Z^\mu + \dots, \quad (21)$$

$$A_{3L}^\mu = \frac{e}{g} A^\mu + \cos \theta_L Z^\mu + \dots, \quad (22)$$

$$A_{3R}^\mu = \frac{e}{g} A^\mu - \tan \theta_L \sin \theta_L Z^\mu + \dots, \quad (23)$$

while the normalization condition ensures that $e^2 = g^2 \sin^2 \theta_L$.

Thus the SM phenomenology is recovered with θ_L playing the role of the standard Weinberg angle¹, and v determined by the Fermi constant. In fact, insertion of (21),

¹ That LR theories predict the SM phenomenology has been shown on very general grounds by Senjanovic [9] who employed the method of Georgi and Weinberg [10].

(22) and (23) in the fermion lagrangian (7) yields the standard model effective lagrangian up to $\mathcal{O}(v^2/w^2)$ corrections. Of course, at low energy, all the effects of the heavy Z' and W_R^\pm are suppressed.

The Higgs sector is very simple, as we only have two neutral scalar fields χ_L^0, χ_R^0 . In the limit $g, \tilde{g} \rightarrow 0$ the mass matrix has a vanishing eigenvalue in the point $\chi_L^0 = v$, $\chi_R^0 = w$. Thus the physical fields are a radial Higgs $\rho = \sqrt{v^2 + w^2}$ and a tangential zero-mass Goldstone boson. For the radial field the mass is determined by the unknown potential $V(\rho)$ and, as for the SM, there are only loose bounds on its value. The cost of LR symmetry breaking seems to be the occurrence of a zero-mass Higgs field; however, for finite gauge couplings $g, \tilde{g} \neq 0$ the $O(2)$ symmetry is not an exact symmetry of the full model, and the tangential “would be” Goldstone boson is expected to acquire a mass.

Until now we have not addressed point (ii) (i.e. the origin of fermion mass matrices), and we avoided to discuss any Higgs–fermion interaction. In the SM the mass of fermions is recovered by insertion of ad hoc interactions. Taking aside composite Higgs theories, which would be compatible with the present LR minimal model, insertion of a fermion–Higgs interaction still remains the simplest way to predict fermion mass matrices. In order to avoid the proliferation of Higgs fields we may build up the four composite matrices $\chi_a \chi_b^\dagger$ with $a, b = L, R$. For each set of ab labels $\chi_a \chi_b^\dagger$ is a matrix since both χ_a and χ_b are doublets according to their definition (4). Moreover, the $O(2)$ symmetry requires that we regard $\Phi \Phi^\dagger \equiv \chi_a \chi_b^\dagger$ as a matrix of matrices. An $O(2)$ invariant interaction term may be written as $\bar{\Psi} \Phi \Phi^\dagger \Psi$. We may also define the adjoint doublets

$$\tilde{\chi}_a = \begin{pmatrix} (\chi_a^0)^* \\ -\chi_a^+ \end{pmatrix}, \quad \tilde{\Phi} = \begin{pmatrix} \tilde{\chi}_L \\ \tilde{\chi}_R \end{pmatrix}, \quad (24)$$

that have the same transformation properties as χ_a and Φ respectively. Thus the more general interaction, invariant under G and $O(2)$ transformations, reads

$$\mathcal{L}_{\text{int}} = -\alpha_1 \bar{\Psi} \tilde{\Phi} \Phi^\dagger \Psi - \alpha_2 \bar{\Psi} \tilde{\Phi} \tilde{\Phi}^\dagger \Psi, \quad (25)$$

where the couplings α_1, α_2 change for different fermionic doublets. If we require exact conservation of the leptonic number, then no Majorana mass term is allowed, and neutrinos are regarded as standard Dirac fermions. The interaction term may be simplified by noticing that $\bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = 0$. Thus we find

$$\mathcal{L}_{\text{int}} = -\alpha_1 \bar{\psi}_L \chi_L \chi_R^\dagger \psi_R - \alpha_2 \bar{\psi}_L \tilde{\chi}_L \tilde{\chi}_R^\dagger \psi_R + \text{h.c.} \quad (26)$$

Then in unitarity gauge, by inserting

$$\chi_a = \begin{pmatrix} 0 \\ \chi_a^0 \end{pmatrix}, \quad \tilde{\chi}_a = \begin{pmatrix} \chi_a^0 \\ 0 \end{pmatrix}, \quad \psi_a = \begin{pmatrix} u_a \\ d_a \end{pmatrix}, \quad (27)$$

the interaction term reads

$$\mathcal{L}_{\text{int}} = -\chi_L^0 \chi_R^0 (\alpha_1 \bar{d}_L d_R + \alpha_2 \bar{u}_L u_R) + \text{h.c.} \quad (28)$$

At low energy the vacuum expectation values of the Higgs fields give mass terms to the fermions. Up and down components of the fermionic doublets get different masses m_u , m_d :

$$m_u = \alpha_2 vw, \quad m_d = \alpha_1 vw. \quad (29)$$

The generalization to the case of three fermionic flavors is straightforward, with the couplings α_1 , α_2 replaced by matrices.

The price we pay is the insertion of a non-renormalizable term in the lagrangian. However the couplings α_i are very small:

$$\alpha_i = \left(\frac{g^2}{4M_{W(R)}} \right) \left(\frac{m_i}{M_{W(L)}} \right) \quad (30)$$

and scale as $1/M_{W(R)}$. The large mass of the top quark suggests that the cut-off scale cannot be much bigger than $M_{W(R)}$, and thus the effective theory makes sense only for energies a little higher than $M_{W(R)}$. However, if θ is very small, that energy scale is considerably higher than any known mass. A full analysis of the phenomenology of the non-renormalizable interactions is beyond the aims of the present paper. On the other hand, we stress that the existence of such non-renormalizable terms in the lagrangian was not required in order to address point (i) (symmetry breaking and the known structure of weak interactions), which is the main aim of this paper. Inclusion of ad hoc terms like \mathcal{L}_{int} is just the simplest way to reproduce the fermion mass matrices.

An open question is the mass of the “would be” tangential Goldstone boson. In principle its value could be evaluated by standard perturbation theory. In a simplified picture we may assume that zero-point energies would contribute² a finite effective potential term $V_b(M) = 3/(64\pi^2)M^4 \ln(M^2/\mu_b^2)$ for any vector boson field with mass M , and a term $V_f(m) = -4/(64\pi^2)m^4 \ln(m^2/\mu_f^2)$ for any fermionic field with mass m . The energies μ_b , μ_f depend on the cut-off scale and can be regarded as free parameters. According to (12), (13) and (29), summing up over all the fermionic masses $m_j = \alpha_j vw = \alpha_j \rho^2 \sin \theta \cos \theta$ and over all the bosonic masses, and assuming $M_Z \approx M_{W(L)} = vg/2$, $M_{Z'} \approx M_{W(R)} = gw/2$, we obtain the following effective potential contribution:

$$\begin{aligned} V_{\text{eff}}(\theta) = & \frac{\rho^4}{64\pi^2} \\ & \times \left[\frac{(3 \cdot 3)g^4}{16} \sin^4 \theta \ln \left(\frac{g^2 \rho^2 \sin^2 \theta}{4\mu_L^2} \right) \right. \\ & + \frac{(3 \cdot 3)g^4}{16} \cos^4 \theta \ln \left(\frac{g^2 \rho^2 \cos^2 \theta}{4\mu_R^2} \right) \\ & \left. - 4\rho^4 \sum_j \alpha_j^4 \sin^4 \theta \cos^4 \theta \ln \left(\frac{\alpha_j^2 \rho^4 \sin^2 \theta \cos^2 \theta}{\mu_j^2} \right) \right], \end{aligned} \quad (31)$$

where the sum over j runs over all the known fermions. The ratio $\rho^4 \alpha_j^4 / g^4 \approx (m_j / M_{W(L)})^4$ is very small and neg-

² The finite contribution may be extracted by the method of Coleman and Weinberg [11].

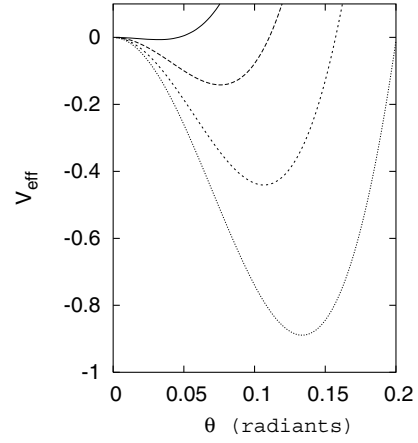


Fig. 1. Finite contributions of zero-point energies to the effective potential according to (32) of the text. A constant has been subtracted in order to set $V_{\text{eff}}(0) = 0$, and an arbitrary scale factor has been inserted. The free parameters a_L , a_R and b are taken to be $a_L = a_R = b = 6.5, 8.0, 9.5, 1.1$ (going from the upper to the lower curve respectively). A minimum is present for a small value of the angle θ which decreases when the free parameters decrease

ligible for almost all fermions. Only the top quark contributes to V_{eff} which reads

$$\begin{aligned} V_{\text{eff}} = & \text{const.} \\ & \times \left[\sin^4 \theta \ln(a_L \sin^2 \theta) + \cos^4 \theta \ln(a_R \cos^2 \theta) \right. \\ & \left. - \xi \sin^4 \theta \cos^4 \theta \ln(b \sin^2 \theta \cos^2 \theta) \right], \end{aligned} \quad (32)$$

where $a_L = (g\rho/2\mu_L)^2$, $a_R = (g\rho/2\mu_R)^2$, $b = \alpha_t^2 \rho^4 / \mu_t^2$ and

$$\xi = \frac{4}{9 \cos^4 \theta_0} \left(\frac{m_t}{M_{W(L)}} \right)^4, \quad (33)$$

with θ_0 fixed at the phenomenological value of θ . Insertion of $M_{W(L)} = 81.5 \text{ GeV}$, $m_t = 181 \text{ GeV}$, $\cos \theta_0 \approx 1$ yields $\xi \approx 11$. The diagram of V_{eff} is reported in Fig. 1 for $a_L = a_R = b$ taken in the range from 0.65 (upper curve) to 1.1 (lower curve). This effective potential term breaks the $O(2)$ invariance, and has an absolute minimum for small values of θ in a broad range of parameters. Detailed calculations are called for in order to compare the predictions of the model with future experimental data on the Higgs mass.

In summary we have shown that the existence of a global $O(2)$ symmetry would give a natural path towards the LR symmetry breaking without requiring complex Higgs sectors. In its minimal version the model only requires two physical neutral Higgs bosons and predicts a phenomenology which is undistinguishable from the SM at the currently reachable energies. Moreover a light Higgs field is predicted whose mass could be evaluated by standard perturbative calculations.

Notes added. A recent paper [12] has addressed the problem of mass generation in the framework of the present

minimal model. That paper does not deal with the symmetry breaking mechanism and barely assumes that the standard electro-weak phenomenology is recovered in the low energy limit. We stress that without any Higgs bidoublet the only non-trivial broken symmetry vacuum would be characterized [3] by a vanishing expectation value $v = 0$ (while $w \neq 0$). Thus any Dirac mass term would be vanishing. Of course soft breaking of symmetry would be a way out (by insertion of ad hoc mass terms in the lagrangian) but that could not be regarded as a genuine spontaneous symmetry breakdown. On the other hand the $O(2)$ symmetry discussed in the present paper seems to be a viable way to generate finite fermion masses without Higgs bidoublets.

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